Problem. I_B is the B-excenter of the triangle ABC and ω is the circumcircle of this triangle. M is the middle of arc BC of ω which doesn't contain A. MI_B meets ω at $T \neq M$. Prove that

$$TB \cdot TC = TI_B^2.$$

After trying to solve the problem on your own, you can find a possible solution on the next page.

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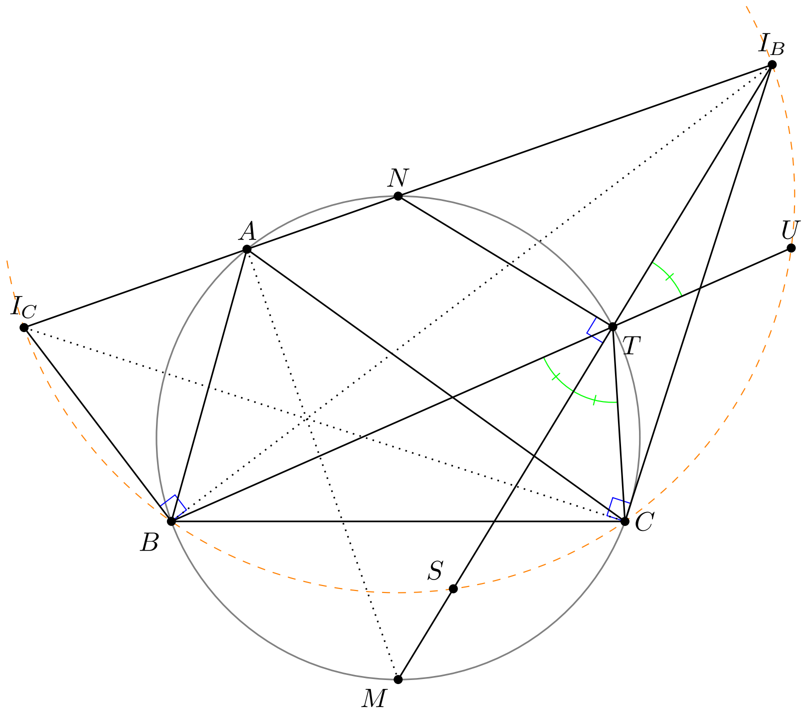

Proof. Let I_C be the excenter opposite to vertex C. We know that BI_B is the angle bisector of $\sphericalangle CBA$ and BI_C is the exterior angle bisector of $\sphericalangle CBA$. Thus, $\sphericalangle I_BBI_C$ is a right angle. In the same way, we get $\sphericalangle I_BCI_C = 90°$. This implies that the Points C, B, I_C and I_B lie on a circle Ω with center on I_BI_C. Since I_BI_C is the exterior angle bisector of $\sphericalangle BAC$, we know that it passes through the midpoint N of arc BC, which contains A. N lies on the perpendicular bisector of BC. Thus, the N is the midpoint of Ω.

Let S and U be the second intersections of I_BT and BT with Ω. Since N and M lie on opposite sides on ω, we know
$$\sphericalangle NTM = 90°.$$
This implies that T is the midpoint of I_BS since N is the center of Ω. Moreover,
$$\sphericalangle UTI_B = \sphericalangle BTM = \sphericalangle MTC = \sphericalangle STC.$$
Here, the second equality follows from the fact that M is the midpoint of arc BC, and thus, MT is the angle bisector of $\sphericalangle BTC$. We get $\sphericalangle NTC = \sphericalangle UTN$, and since U and C lie on a circle with center N, they are symmetric with respect to the perpendicular bisector NT of SI_B. Thus, $TU = TC$. Putting everything together, we conclude
$$TB \cdot TC = TB \cdot TU = TS \cdot TI_B = TI_B^2.$$

q.e.d.